The Coulomb Critical Taper theory applied to gravitational instabilities

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Régis Mourgues
Cynthia Garibaldi
1. The Critical Coulomb wedge theory

Modified after Dahlen (1990)
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Wedge growth
Thrusting

Steepening until the critical taper is attained

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Wedge growth
Thrusting

Steepening until the critical taper is attained

Sliding along the basal detachment

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1. The Critical Coulomb wedge theory

General solution (Davis et al., 1983)

Surface of the wedge

Basal detachment
1. The Critical Coulomb wedge theory

General solution (Davis et al., 1983)

\[ \alpha + \beta \sim \rho, \lambda, \phi, \Psi, H \]
1. The Critical Coulomb wedge theory

Case of a noncohesive wedge (Dahlen, 1984)

Exact solution: $\alpha + \beta = \Psi_b - \Psi_o$
1. The Critical Coulomb wedge theory

Case of a noncohesive wedge (Dahlen, 1984)

Exact solution: \( \alpha + \beta = \Psi_b - \Psi_o \)

- Mohr-Coulomb criterion of deformation
- Wedge everywhere on the verge of failure
1. The Critical Coulomb wedge theory

- Mohr-Coulomb criterion of deformation:

\[ \tau = \mu \sigma_n + C \]

- Wedge everywhere on the verge of failure

Stress : deformation
1. The Critical Coulomb wedge theory

Accretionary wedges

\[ \phi = 39^\circ \]
\[ \phi_b = 30^\circ \]
\[ \lambda = \lambda_b = 0 \]

Modified after Dahlen (1984)
1. The Critical Coulomb wedge theory

Accretionary wedges

Modified after Dahlen (1984)

Sub-critical wedges: formation of thrusts
1. The Critical Coulomb wedge theory

**Accretionary wedges**

- **Critical wedge theory**
  - **Critical taper**
  - \( \phi = 39^\circ \)  
  - \( \phi_b = 30^\circ \)  
  - \( \lambda = \lambda_b = 0 \)

- **Stable supercritical wedges**: sliding on basal detachment

- **Sub-critical wedges**: formation of thrusts

*Modified after Dahlen (1984)*
1. The Critical Coulomb wedge theory

**Accretionary wedges**

**Unstable wedges**: shallow slumping + normal faulting

**Stable supercritical wedges**: sliding on basal detachment

**Sub-critical wedges**: formation of thrusts

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**Modified after Dahlen (1984)**
1. The Critical Coulomb wedge theory

Accretionary wedges

Modified after Dahlen (1984)

Dahlen (1990)
1. The Critical Coulomb wedge theory

Extensional wedges

\( \phi = 39^\circ \)  
\( \phi_b = 30^\circ \)  
\( \lambda = \lambda_b = 0 \)

Modified after Xiao et al. (1991)
1. The Critical Coulomb wedge theory

**Extensional wedges**

**Sub-critical wedges:** normal faulting

Modified after Xiao et al. (1991)
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**Extensional wedges**

**Sub-critical wedges:** normal faulting

**Stable wedges:** sliding on basal detachment

Modified after Xiao et al. (1991)
1. The Critical Coulomb wedge theory

**Extensional wedges**

*Sub-critical wedges:* normal faulting

*Stable wedges:* sliding on basal detachment

- **Active** extensional setting
- Basal shear stress towards the thicker part

Modified after Xiao et al. (1991)
1. The Critical Coulomb wedge theory

**Extensional wedges**

\[ \phi = 39° \]
\[ \phi_b = 30° \]
\[ \lambda = \lambda_0 = 0 \]

*Modified after Xiao et al. (1991)*
1. The Critical Coulomb wedge theory

**Extensional wedges**

![Graph showing surface slope and basal dip relationship](image)

*Modified after Xiao et al. (1991)*

\[ -\alpha \text{ increases } \beta \text{ constant} \]
1. The Critical Coulomb wedge theory

**Extensional wedges**

Modified after Xiao et al. (1991)

$-\alpha$ increases  
$\beta$ constant

Faulting until stable configuration
1. The Critical Coulomb wedge theory

**Extensional wedges**

Modified after Xiao et al. (1991)
What if there is no external force (other than gravity)?

- Gravitational spreading
- Basal shear stress towards the thinner part
2. The theory adapted to gravitational instabilities

- Noncohesive material on the verge of failure
2. The theory adapted to gravitational instabilities

- Noncohesive material on the verge of failure
- System subjected to pore-fluid pressure
2. The theory adapted to gravitational instabilities

- Noncohesive material on the verge of failure
- System subjected to pore-fluid pressure
- No downslope buttress
2. The theory adapted to gravitational instabilities

A part of the total stresses is supported by the fluid

**Effective stresses**

\[ \sigma' = \sigma - P_f \]
2. The theory adapted to gravitational instabilities

Equations of equilibrium:

\[ \sigma'_{zz} = (1-\lambda^*) \rho gz \cos \alpha \]

\[ \sigma'_{xz} = \rho gz \sin \alpha \]
2. The theory adapted to gravitational instabilities

Equations of equilibrium:

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Fluid overpressure ratio:

\[ \lambda^* = \frac{P_{ov}}{\rho gz \cos \alpha} \]
2. The theory adapted to gravitational instabilities

Equations of equilibrium:

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Fluid overpressure ratio:

\[ \lambda^* = \frac{P_{ov}}{\rho gz \cos \alpha} \]

\[ \lambda^* = 0 \text{ in hydrostatic equilibrium} \]
2. The theory adapted to gravitational instabilities

Expression of $\sigma'_{xx}$:

2 values of $\sigma'_{xx}$

Extensional & Contractional states of stress
2. The theory adapted to gravitational instabilities

Expression of $\sigma'_{xx}$:

2 values of $\sigma'_{xx}$

Extensional & Contractional states of stress

*Rankine states of equilibrium*
2. The theory adapted to gravitational instabilities

**Expression of $\sigma'_{xx}$:**

Without downslope resistance

*Only extensional state of stress within the wedge*
2. The theory adapted to gravitational instabilities

Expression of $\sigma_{xx}'$:

Without downslope resistance

Only extensional state of stress within the wedge

$$\sigma_{xx}' = (2Y-1)\sigma_{zz}'$$

with $Y = \frac{1 - \sin \sqrt{1 - FS^2}}{\cos^2 \phi}$
2. The theory adapted to gravitational instabilities

The factor of safety $FS$:

$$FS = \frac{\tan \alpha}{(1 - \lambda^*) \tan \phi}$$
2. The theory adapted to gravitational instabilities

The factor of safety $FS$:

$$FS = \frac{\tan \alpha}{(1 - \lambda^*) \tan \phi}$$

- Corrected from the fluid overpressure
- $FS > 1$: unstable slope, shallow landsliding
2. The theory adapted to gravitational instabilities

The effective basal friction $\mu'_b$:

Sliding = low friction on the basal detachment:

$$\mu_b(1-\lambda^*_b) < \mu(1-\lambda^*)$$
2. The theory adapted to gravitational instabilities

The effective basal friction $\mu'_b$:

Sliding = low friction on the basal detachment:

$$\mu_b(1-\lambda^*_b) < \mu(1-\lambda^*)$$

After expressing $\sigma'_{xz}$ and $\sigma'_{zz}$ on the detachment:

$$\lambda^*_b = 1 - (1 - \lambda^*) \frac{E_2}{\mu_b E_1}$$
2. The theory adapted to gravitational instabilities
2. The theory adapted to gravitational instabilities

System subjected to gravity only: 3 domains
2. The theory adapted to gravitational instabilities

System subjected to gravity only: 3 domains
2. The theory adapted to gravitational instabilities

*Alternative expression of $\mu'_b$:*

Dahlen’s definition:

\[ \mu'_b = \mu_b \frac{1 - \lambda_b}{1 - \lambda} \]
2. The theory adapted to gravitational instabilities

Alternative expression of $\mu'_b$:

Dahlen’s definition:

$$\mu'_b = \mu_b \frac{1 - \lambda_b}{1 - \lambda}$$

$\text{xz coordinate system (surface)}$

$\sigma_{zz}$ independent of $\lambda$
2. The theory adapted to gravitational instabilities

**Alternative expression of \( \mu'_b \):**

Dahlen’s definition:

- \( xz \) coordinate system
- \( \sigma_{zz} \) independent of \( \lambda \)

But \( \mu'_b \) **dependent on \( \sigma'_{z_1z_1} \) (varying with \( \lambda \))**
Alternate expression of $\mu'_b$:

Dahlen’s definition:

- xz coordinate system
- $\sigma_{zz}$ independent of $\lambda$

But $\mu'_b$ dependent on $\sigma'_{z'z'}$ (varying with $\lambda$)

Definition in the $x'z'$ coordinate system (detachment)

$$\lambda^*_b = E_1 + \lambda^* - \frac{E_2}{\mu_b}$$
2. The theory adapted to gravitational instabilities

Alternative expression

\[ \beta = 0^\circ \\
\mu = 32^\circ \\
\mu_b = 22^\circ \\
\lambda = 0 \]
2. The theory adapted to gravitational instabilities

- Negligible differences for compressive wedges
- Higher critical gravitational sliding limit

Alternative expression
3. Experimental modelling

- Experimental verification of the theory

- Previous works:

  Mostly compressive settings
  Fluid pressure not taken into account
3. Experimental modelling

Experimental set-up

<table>
<thead>
<tr>
<th>Material</th>
<th>Grain size (μm)</th>
<th>Bulk density (kg/m³)</th>
<th>Angle of internal friction (°)</th>
<th>Coefficient of internal friction µ</th>
<th>Permeability (Darcy)</th>
<th>Cohesion (Pa)</th>
</tr>
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<tbody>
<tr>
<td>Coarse sand (cover)</td>
<td>300</td>
<td>1600</td>
<td>34</td>
<td>0.67</td>
<td>90</td>
<td>0</td>
</tr>
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<td>Glass microbeads (décollement)</td>
<td>200-300</td>
<td>1600</td>
<td>24</td>
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3. Experimental modelling

Experimental set-up

A

coarse sand

glass microbeads

air input

location of pressure profile

sand wedge

detachment surface

reservoir

adjustable basal sieve

angle α

β

H

H'

B

\[ K_M = 15D \]

\[ K_s = 90D \]

H = 2 cm

\[ \alpha = 15^\circ \]

C

\[ \lambda_b^* \]

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3. Experimental modelling

**Experimental set-up**

- No downslope buttress
- Adjustable basal and surface slopes
- $\lambda^*_b$ constant along the detachment
3. Experimental modelling

**Experimental procedure**

\[ \alpha = 15^\circ \; ; \; \beta = 10^\circ \]

- Increasing air pressure
3. Experimental modelling

**Experimental procedure**

- Increasing air pressure
- Measurements of the critical fluid pressure when sliding

\[ \alpha = 15^\circ; \beta = 10^\circ \]
3. Experimental modelling

Results

- Experimental data
- Model I solution
- Alternative solution
Good agreement between theory and experience
4. Discussion

Good agreement between theory and experience

However, difficulties to discriminate (I or II?)
4. Discussion

Good agreement between theory and experience

However, difficulties to discriminate (I or II?)

Experimental uncertainties:
4. Discussion

Good agreement between theory and experience

However, difficulties to discriminate (I or II?)

Experimental uncertainties:

- Shape of the detachment
4. Discussion

Good agreement between theory and experience

However, difficulties to discriminate (I or II?)

Experimental uncertainties:

- Shape of the detachment
- Permeabilities
4. Discussion

Good agreement between theory and experience

However, difficulties to discriminate (I or II?)

Experimental uncertainties:
- Shape of the detachment
- Permeabilities
- Pressure losses
4. Discussion

Good agreement between theory and experience

However, difficulties to discriminate (I or II?)

Experimental uncertainties:

- Shape of the detachment
- Permeabilities
- Pressure losses
- Air moisture
4. Discussion

Good agreement between theory and experience

*However, difficulties to discriminate (I or II?)*

Experimental uncertainties:
- Shape of the detachment
- Permeabilities
- Pressure losses
- Air moisture

More models needed (low $\alpha$)
5. Applicability to natural examples

- Not restricted to accretionary prisms

weak décollement and no downslope buttress

A- Large submarine slumping

Storegga slide, Norway
5. Applicability to natural examples

- Not restricted to accretionary prisms

weak décollement and no downslope buttress

A- Large submarine slumping

Rising fluids

Storegga slide, Norway
5. Applicability to natural examples

A- Large submarine slumping

Modified after Kvalstad et al. (2005)
5. Applicability to natural examples

A- Large submarine slumping

Modified after Kvalstad et al. (2005)
5. Applicability to natural examples

B- Transform margins

Image Google Earth
5. Applicability to natural examples

B- Transform margins

Image Google Earth
5. Applicability to natural examples

B- Transform margins

Demerara Plateau

Image Google Earth
5. Applicability to natural examples

B- Transform margins

Pattier et al. (2013)
5. Applicability to natural examples

C- Onshore Landslides

Waitawhiti, New Zealand
5. Applicability to natural examples

C- Onshore landslides

Lacoste et al. (2009)
5. Conclusions

- Critical Coulomb wedge theory applicable to systems subjected to gravitational forces only
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- Experimental verification of the theory
5. Conclusions

- Critical Coulomb wedge theory applicable to systems subjected to gravitational forces only

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- 2 different solutions but insufficient results to validate either one or the other: more experiments are required...
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- Critical Coulomb wedge theory applicable to systems subjected to gravitational forces only

- Experimental verification of the theory

- 2 different solutions but insufficient results to validate either one or the other: more experiments are required...

- Potential applications to natural systems: passive margins, landslides